# Trigonometry (1/5): Introduction and Overview Introduction to Engineering Mathematics

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### The unit circle

- The circle of radius 1 in the *xy*-plane, centered on the origin.
- Equation:  $x^2 + y^2 = 1$
- Four quadrants: *I*, *II*, *III*, *IV*



## Example

# If $P(\sqrt{3}/2,y)$ is a point on the unit circle, find the value of y.

Angles and points on the unit circle

- Each point P(x, y) defines an angle θ measured from the positive x-axis in counterclockwise direction.
- Angles measured in degrees or radians.
  - Value of  $\theta$  in radians: length of arc subtended by  $\theta$  (length of the red segment)



## Converting between angles and radians

General formula to convert between degrees and radians:

degrees 
$$\xrightarrow{\times \frac{\pi}{180}}_{\times \frac{180}{\pi}}$$
 radians

	Degrees	Radians
Full circle	$360^{\circ}$	$2\pi$
Half circle	$180^{\circ}$	$\pi$
Quarter circle	$90^{\circ}$	$\pi/2$



# Negative angles

Measured from the positive x-axis, in clockwise direction.



#### Adding $2\pi$ to an angle

- Point P is determined by the angle  $\theta$ .
- P stays same when adding  $\pm 2\pi$  to  $\theta$ .
- $\Rightarrow \mathsf{All} \text{ angles } \theta + 2k\pi \text{ with } k \in \mathbb{Z} \text{ give the same point } P.$

**Principal angle**:  $\theta$  such that  $-\pi < \theta \leq \pi$ .



## Finding the coordinates of a point

Given an angle  $\theta$ , find the coordinates of P(x, y).



## Finding the coordinates of a point

Slightly more involved case:

$$\bullet \theta = \pi/4$$



### Important angles

Angle	x-coordinate	y-coordinate
0	1	0
$\pi/6$	$\sqrt{3}/2$	1/2
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$	1/2	$\sqrt{3}/2$
$\pi/2$	0	1
$\pi$	-1	0
$2\pi$	1	0

# Trigonometric functions as coordinates

Let  $\theta$  be an angle with point P(x, y).

Name	Notation	Definitior
Cosine	$\cos  heta$	x
Sine	$\sin  heta$	y
Tangent	an heta	$\frac{\sin\theta}{\cos\theta}$
Cotangent	$\cot  heta$	$\frac{\cos\theta}{\sin\theta}$
Cosecant	$\csc  heta$	$\frac{1}{\sin\theta}$
Secant	$\sec \theta$	$\frac{1}{\cos\theta}$



Example

Given that 
$$heta=rac{\pi}{6}$$
, find the values of all 6 trigonometric functions.

### Fundamental identity

- P(x,y) is on the unit circle:  $x^2 + y^2 = 1$
- Put  $x = \cos \theta$  and  $y = \sin \theta$  to obtain the fundamental identity:



#### Aside: notation

Be very careful when you see  $\sin^k \theta$ .

• Positive exponent (**power**):

$$\sin^k \theta = (\sin \theta)^k.$$

• Negative exponent -1 (inverse function):

$$\sin^{-1} y = \arcsin y.$$

#### Fundamental identity: consequences

$$1 + \tan^2 \theta = \sec^2 \theta \qquad \qquad 1 + \cot^2 \theta = \csc^2 \theta$$

Example

Suppose 
$$\cos \theta = -\frac{4}{5}$$
 and  $\theta$  is in quadrant III. Find  $\sin \theta$  and  $\tan \theta$ .

# Periodicity of sine and cosine

• Sine and cosine are  $2\pi$ -periodic:

$$\begin{aligned} \sin(\theta\pm 2\pi) &= \sin\theta\\ \cos(\theta\pm 2\pi) &= \cos\theta \end{aligned}$$

• The tangent is  $\pi$ -periodic:

$$\tan(\theta \pm \pi) = \tan \theta$$

Example: Compute  $\tan\left(\frac{8093\pi}{4}\right)$ 

