Theory of equations (1/2): Working with polynomials Introduction to Engineering Mathematics

Prof. Joris Vankerschaver

Overview

- Polynomial long division
- Synthetic division of polynomials

Reminder: what is a polynomial?

Polynomial of degree n:

$$P(x)=a_nx^n+a_{n-1}x^{n-1}+\cdots+a_1x+a_0$$

with

- x: the unknown
- a_0, \ldots, a_n : the coefficients (in \mathbb{C} or \mathbb{R})

Reminder: Euclidian division of numbers

For every two integers p and d, we can write the fraction p/d as

$$\frac{p}{d} = q + \frac{r}{d},$$

with q the **quotient** and r the **remainder**. In other words

$$p = qd + r.$$

The numbers q and r can be found by *long division*.

Properties:

- The remainder r is always smaller than d.
- The quotient q and remainder r are unique.

If we divide 20 by 7, we get

$$\frac{20}{7} = 2 + \frac{6}{7},$$

or $20 = 2 \times 7 + 6$.

Therefore:

- Quotient: q = 20
- Remainder: r = 6.

Euclidian division of polynomials

For every two polynomials P(x) and D(x), we can write the rational function P(x)/D(x) as

$$\frac{P(x)}{D(x)} = Q(x) + \frac{R(x)}{D(x)},$$

with P(x) the **quotient** and R(x) the **remainder**. In other words

$$P(x) = Q(x)D(x) + R(x).$$

The polynomials Q(x) and R(x) can be found by *long division*. Properties:

- The degree of R(x) is always smaller than the degree of D(x).
- The quotient Q(x) and remainder R(x) are unique.

If we divide $x^2 + 1$ by x - 1, we get:

$$\frac{x^2 + 1}{x - 1} = \frac{x^2 - 1 + 2}{x - 1}$$
$$= \frac{x^2 - 1}{x - 1} + \frac{2}{x - 1}$$
$$= x + 1 + \frac{2}{x - 1}$$

Therefore, the quotient is Q(x) = x + 1 and the remainder is R(x) = 2.

Polynomial long division

If $P(x) = 3x^4 - x^3 + 2x^2 - 2x - 1$ and D(x) = x + 2, find the quotient and the remainder after dividing P(x) by D(x).

Algorithm:

- 1 Divide leading term by leading term and write down result
- 2 Multiply result by divisor and transfer the result to the left
- 3 Subtract
- Orop next term
- **6** Repeat steps 1-4.

Divide
$$P(x) = 3x^4 - x^3 + 2x^2 - 2x - 1$$
 by $D(x) = x + 2$.

Synthetic division

Algorithm:

- $\ensuremath{\textcircled{}}$ Write down coefficients of P(x) on top
- **2** Write down coefficients of -D(x) (except the first) on left
- 3 Lower first coefficient
- 4 Multiply and put result back in the table
- 6 Add coefficients in next column
- 6 Repeat step 4-5 until done
- Ø Determine quotient and remainder

If $P(x) = x^3 - 12x^2 - 42$ and D(x) = x - 3, find the quotient and the remainder after dividing P(x) by D(x).

Special case

$$\frac{x^n - 1}{x - 1} = x^{n - 1} + x^{n - 2} + \dots + x + 1.$$

Resources

Step-by-step walkthroughs of both algorithms (from the 2021-22 version of this course):

- Polynomial long division: https://youtu.be/RyRqUg5oycE?t=499
- Synthetic division: https://youtu.be/NqQeMfGEzk4

These videos can be found on Ufora as well.